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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

34. Proposed by R. H. YOUNG, West Sunbury, Pennsylvania.

Prove (1) that $\frac{n(n+1)(2n+1)}{6}$ is a whole number for all values of n; and (2) prove that $\frac{n(n-1)(n+1}{24}$ is a whole number when n is odd.

I. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tennessee.

(1). $1^2+2^2+3^2+4^2+\dots\dots+n^2=\frac{n(n+1)(2n+1)}{6}=$ a whole number for all integral values of n.

(2). Let $n=2m+1=$ an odd number for all integral values of m.

$$\therefore \frac{(n-1)n(n+1)}{24}=\frac{m(m+1)(2m+1)}{6}=\text{same as (1)}.$$

II. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

As n and $n+1$ are consecutive numbers, one of them must be even and so divisible by two. But n must be of the form of $3p$, $3p+1$, or $3p+2$. If of the form of $3p$, it is divisible by three; if of the form $3p+2$, then $n+1$ or $3p+3$ is divisible by three; if of the form $3p+1$, then $(2n+1)$ becomes $6p+3$, and is divisible by three. Hence $n(n+1)(2n+1)$ is divisible by twice three, or six, whatever the value of n is.

2. $(n-1)n(n+1)$ of which the middle one is odd. One of every three consecutive numbers is always divisible by three: one of two consecutive *even* numbers is always divisible by four and the other by two. Hence $(n-1)n(n+1)$, when n is odd, is always divisible by $2\times3\times4$ or 24.

Also solved by *O. W. ANTHONY, M. A. GRUBER, EDGAR KESNER, E. W. MORRELL, J. SCHEFFER, E. L. SHERWOOD, B. F. YANNEY*, and *G. B. M. ZERR.*

35. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science in Texarkana College, Texarkana, Arkansas-Texas.

Decompose into the sum of two squares the number $13^2.61^3$.

I. Solution by E. L. SHERWOOD, A. M., Professor of Mathematics in Mississippi Normal College, Houston, Miss., and E. W. MORRELL, Department of Mathematics in Montpelier Seminary, Montpelier, Vermont.

$$13^2.61^3=13^2.61^2.61=13^2.61^2(5^2+6^2)=13^2.61^2.5^2+13^2.61^2.6^2.$$

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Put $13^2.61^3=(p^2+q^2)^2(m^2+n^2)^3$, in which $p=3$, $q=2$, $m=6$, $n=5$. By decomposing into the sum of two squares, we find

$$(p^2+q^2)^2(m^2+n^2)^3=[m(p^2+q^2)(m^2-3n^2)]^2+[n(p^2+q^2)(3m^2-n^2)]^2=$$
$$[m(m^2+n^2)(p^2+q^2)]^2+[n(m^2+n^2)(p^2+q^2)]^2=$$
$$[m(p^2-q^2)(m^2-3n^2)\pm 2npq(3m^2-n^2)]^2+$$
$$[n(p^2-q^2)(3m^2-n^2)\mp 2mpq(m^2-3n^2)]^2=$$
$$[m(m^2+n^2)(p^2-q^2)\pm 2npq(m^2+n^2)]^2+[n(m^2+n^2)(p^2-q^2)\mp 2mpq(m^2+n^2)]^2,$$

making *six* sets of the sum of two squares.

Substituting the respective values of p, q, m, and n, we have

$$13^2.61^3=3042^2+5395^2=4758^2+3965^2=3810^2+4883^2$$
$$=6150^2+733^2=5490^2+2867^2=1830^2+5917^2.$$

Solved with these six sets of values by *A. H. BELL*, and with the five sets last in order by the *PROPOSER*. Also solved by *J. H. DRUMMOND*, *C. D. SCHMITT*, and *B. F. YANNEY*.

36. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find the first six integral values of n in $\frac{n(n+1)}{2}=\square$.

I. Solution by Professor J. SCHEFFER, A. M., Hagerstown, Maryland, and O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland.

We have $n^2+n=2y^2$. Putting $n=\frac{t-1}{2}$, we obtain $t^2-8y^2=1$. Since $t=3$, $y=1$, satisfy this equation, we have $t=\frac{1}{2}[(3+2\sqrt{2})^m+(3-2\sqrt{2})^m]$, where for m successive integral numbers must be chosen. The required values of n we then obtain from the relation $n=\frac{t-1}{2}$.

For $m=1$, 2, 3, 4, 5, 6, in succession, we find in order the corresponding values of $t=3$, 17, 99, 577, 3363, 19601, and $n=1$, 8, 49, 288, 1681, 9800.

II. Solution by A. H. BELL, Hillsboro, Illinois.

Let $\frac{n(n+1)}{2}=\square=y^2$ say ; then clearing of fractions, multiplying by 4, and adding 1 to both members, etc., $(2n+1)^2=8y^2+1=\square=x^2$ say.

$\therefore$ $n=\frac{x-1}{2}$. Again $x^2-8y^2=1$. The 1st convergent for the $\sqrt{8}=\frac{3}{1}\frac{=x}{=y}$ or the solution of this celebrated equation and the value of x and y can be found, on page 53, Vol. I. of MONTHLY.

The general value for x is $x_{n+1}=2x_1\times x_n-x_{n-1}$, hence $x_0=1$, $x_1=3$, $x_2=6\times 3-1=17$, $x_3=6\times 17-3=99$, $x_4=6\times 99-17=577$, $x_5=6\times 577-99=3363$, and $x_6=19601$, etc.

$\therefore$ The required values of $n=1$, 8, 49, 288, 1681, 9800, etc.

III. Solution by the PROPOSER.

When $\frac{n(n+1)}{2}=\square$, one of the factors, n and $n-1$, is a square and the